Cyclic Components in the Length of Solar Cycle

G.K. Rangarajan
L.M. Barreto*

Resumo
A série temporal das durações do ciclo solar, cobrindo mais de 22 séculos, é decomposta em três ciclos com periodicidades aproximadas de 80, 50 e 240 anos, e são indicadas as variações temporais em amplitude. Sugere-se que esses componentes cíclicos podem ser úteis para melhor estabelecer a interrelação entre a atividade solar e o clima. Uma extrapolação da série temporal de 25 anos até 2015, indica que nos próximos anos, a duração do ciclo solar tenderá a mostrar um acréscimo marginal até 2010. A duração do ciclo solar derivada dos dados extrapolados parece ser consistente com aquela derivada da média anual do número de manchas solares, após 1700 AD.

Abstract
The time series of the length of solar cycle covering more than 22 centuries is decomposed into three major cyclic components with associated periodicities of ≈80, ≈50 and ≈240 years, and the time variations in their amplitude are indicated. It is suggested that these cyclic components could be useful to establish better the interrelationship between solar activity and weather/climate. Extrapolation of the time series by 25 years up to 2015 indicates that in the coming years, the length of the solar cycle will tend to show marginal increase up to 2010. Length of the solar cycle derived from proxy data seems to be consistent with that derived from annual mean sunspot numbers after 1700 AD.

* Observatorio Nacional, São Cristovão, Rio de Janeiro, Brazil.
Introduction

Zhou and Butler (1998) recently carried out a statistical study of the relationship between the length of the solar cycle and the tree-ring index values. For their analysis, they used the list of the years of solar maxima between 284 BC and 1700 AD, compiled by Schouve (1955), based on a variety of historical sources including observed sunspot numbers and visual aurorae. Between 1700 and 1990, the available annual mean sunspot numbers were used. After the discovery by Friis-Christensen and Lassen (1991) that shorter lengths of the solar cycle leads to higher temperatures over the northern hemisphere in the last 130 years and the subsequent work of Lassen and Friis-Christensen (1995), this parameter has assumed importance. According to Zhou and Butler (1998), solar cycle length is not at variance with other solar activity parameters. The time variation in the length of the solar cycle was provided as Figure 1 in their paper. This figure suggests that (i) there is no apparent trend in the length of the solar cycle and (ii) there could be hidden quasi-periodic fluctuations which may have special significance in solar-climate studies.

Isolating distinct components of variation in a noisy time series has been made fairly easy since the work of Vautard and Ghil (1989) and Vautard et al. (1992). In these publications, they have highlighted how the technique of Singular Spectrum Analysis (SSA) provides quantitative and qualitative information about the deterministic and stochastic parts of a time series. SSA extracts important components of the variability even when the system is non-stationary. It was also shown how this method, which is a form of Principal Component Analysis applied to the lag-correlation structure of the time series, succeeds in isolating trends, quasi-periodic fluctuations and noise through the generation of suitable data adaptive filters and how the original time series could be closely approximated by the addition of only few principal components.

Long term variations in climate change have been linked in some form, hitherto not very specifically, to solar activity. Even the solar constant has been shown to be somewhat variable! We believe that the length of the solar cycle will be one of the more useful parameters in the studies relating Solar activity to the Earth’s climatic changes, though we concede that the climate is a complicated system with many interacting factors. It is also possible that the effects of solar activity on climate could well be non-linear and approaches through statistical methodologies could be prone to error. However, we are emboldened from the results of the present analysis from the fact that the identified periodicities by the Singular Spectrum Analysis, have been detected in solar activity parameters in the past.
Figure 1. Singular spectrum derived from the time series of the length of solar cycle between 284 BC and 1990 AD (given by Zhou and Butler, 1998). The order M or the maximum length of the lags auto-correlations utilized was 100, but it is noticed that beyond about 20, all values are 0. Singular spectrum is defined as the square root of the M eigen values.
Some advantages of the technique of SSA

Illustrative applications of this methodology could be seen in Rangarajan and Iyemori (1998). Rangarajan and Araki (1997) have repeated, for the benefit of geophysicists, the formulation of Vautard and Ghil (1989) indicating how a time series is used to generate a trajectory matrix, its variance-covariance matrix and how to extract the eigen values and eigen vectors of the variance-covariance matrix. It is shown that as the matrix is positive symmetric Toeplitz type, all the eigen values would be real with number of non-zero eigen values corresponding to the number of independent variables in the system. If we plot the square root of the eigen values in descending order—the so called singular spectrum—the insignificant parts of the eigen values will define a noise floor close to zero (see Sharma et al., 1993). This process enables us to isolate the significant oscillations from insignificant background noise with appropriate confidence levels associated with the eigen values themselves. Interestingly, when eigen values occur in pairs (nearly equal values), they are indicative of quasi-periodic oscillations present in the original time series. An illustrative application of this methodology could be seen in Rangarajan and Iyemori (1998).

In this note, we show that the time series of the length of the solar cycle, over 2275 years, can be conveniently decomposed into only three significant oscillations with additional three more of marginal significance. Also, we provide the time variations of these components for the people dealing with sun-weather relationships, as we are interested mainly in geomagnetism and magnetospheric physics.

Data Analysis

Figure 1 of Zhou and Butler (1998) was digitized to generate 2275 annual values of the length of the solar cycle between 284 BC and 1990 AD. Auto-correlations up to 100 lags are computed and the eigen values of the Toeplitz matrix generated from this series of auto-correlations are determined together with the corresponding eigen vectors. The elements of the eigen vectors associated with only the significant eigen values are then used as data adaptive filters to construct the individual principal components. The singular spectrum, defined as the square root of the eigen values, is shown in Figure 1. Six principal components are shown in Figure 2. The adequacy of these six components to reconstitute the original time series is demonstrated in Figure 3 which shows: (i) the original time series provided by Zhou and Butler (1998), (ii) the reconstructed time series from SSA using the six principal components only and (iii) the difference between the two series.
Time variations amplitude in the first six significant principal components of the analysed time series. Approximate periodicity is indicated. The percentage values given in brackets refer to the relative variance accounted for by the component. Note that the ordinate scales are magnified by a factor of 10.
Discussion

The major advantage of the SSA methodology is the fact that we can estimate from the singular spectrum the number of significant components and the variance contributed by each of these components to the total variance of the time series. It is generally accepted that eigen values close to zero define a ‘noise floor’ in the spectrum and only terms above this level need to be taken into account (see Sharma et al., 1993 for instance). Also, if eigen values appear as closely matched pairs together with the fact that the corresponding eigen vectors are in phase quadrature, then it is indicative of the presence of quasi-periodic fluctuations (Vautard et al., 1992). From Figure 1, we can establish that the first 6 components together can account for more than 98 percent of the total variability, and of them, the first two pairs represent quasi-periodic fluctuations. Components are identified by means of their order M.

Figure 2 shows the time variations of the six components in decreasing order of their relative contribution to the total variance. A signal of ≈80-year periodicity (estimate of the period by MEM of the principal component leads to a value of 78.5 years) is the most dominant of the variations. The presence of the Gleisberg cycle in solar activity is, of course, fairly well established and its influence has been noted, among others, in geomagnetic activity, for example by Feynman and Crooker (1978). This particular component itself introduces a change in the length of the solar cycle by as much as 1.5 years from the average in some periods and practically causes no change in others. The second most important oscillation has a periodicity of 48.5 years, followed by one with a quasi-periodicity of 240 years, superposed with a higher frequency component, with periodicity of ≈40 years, which could not be isolated. An underlying ≈200-year oscillation in solar activity levels was indeed indicated by Zhou and Butler (1998); here we are able to precisely quantify its existence and the intermittence in the significance of its amplitude. Three other oscillations which together account for about 10 percent of the remaining variability have associated periodicity of 37.3, 32.5 and 28.8 years, as determined from the spectra of the individual components.

When these six components are added, together with the average value of the length of the solar cycle (11.23 years), we are able to reproduce the original time series, as shown in Figure 3. The noisy part, not accounted for, works out to less than 2 percent of the total variance and, interestingly, there is only a very short interval between 1200 and 1500 AD when the noise is significantly higher compared to other times. Another noteworthy feature of the ‘noise’ as well as the six isolated components is the fact that there is an excellent continuity in the time series between the epochs prior to and after 1700 AD, when the annual mean sunspot number series began. It appears that at least in so far as the length of the solar cycle
Figure 3. The original time series (digitized data), the time series derived from the addition of the six components and the mean value and difference between the two time series.
is concerned, these are quite well determined from 284 BC, as suggested by Zhou and Butler (1998).

These features clearly suggest that the isolated cyclic components are statistically significant, particularly because they are derived by adaptative filtering using the eigen vectors corresponding to the most significant eigen values indicated in the singular spectrum shown in Figure 1. An additional measure of the associated confidence of the quasi-periodic oscillation, in decreasing order of importance, is the variance accounted by that component, given in brackets next to the identified periodicity in Figure 2.

We next attempt an extrapolation of the time series of Zhou and Butler (1998) by 25 years, adapting a methodology based on SSA and AutoRegressive (AR) modeling. Keppenne and Ghil (1992) demonstrated the intrinsic ability of the method of adaptive filtering through SSA, and subsequent prediction through low order AR process to forecast the southern oscillation index, three years ahead. Pendland et al. (1991), earlier, showed that the spectrum of a time series derived from the cumulative addition of the spectra of the individual principal components of SSA leads to much better resolution and is substantially noise-free even when very low order AR process is used, as compared to the spectrum derived directly from the data. Using the AR coefficients \( a_k \) \((k=1, 2, \ldots, M)\), the time series \( X \) can be extrapolated based on past observations as

\[
X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_M X_{t-M} + e_t
\]

where \( e \) is the residual noise. Keppenne and Ghil (1992) suggested that the individual principal components can be extrapolated beyond the data length by low order AR coefficients, and all predicted values can then be combined to generate the amplitude and shape of the time series in the region of extrapolation. Rangarajan (1998) suggested some improvement to the methodology using weighted averages of several sequences of prediction by considering different segments of actually observed data. Following this method, we attempt to predict the length of the solar cycle for 25 years ahead of 1990. The actual observations from 1916 to 1990 together with the anticipated extrapolation, through the approach elaborated above, are shown in Figure 4. It is seen that up to about 2015 AD, the length of the solar cycle changes over a fairly small range of amplitude with a tendency for increase in the length till 2010.
Figure 4. The original time series of the length of the solar cycle between 1916 and 1990 and extrapolation based on SSA and AR modeling up to 2015 (to cover 100 years altogether).
In conclusion, we show that the extremely long time series of the length of the solar cycle provided by Zhou and Butler (1998) basically consist of three significant quasi-periodic oscillations with periods of ≈80, ≈50 and ≈240 years, together with three others of marginal significance. These periodic variations show change in amplitude and intermittence in appearance over the interval of time considered. It is suggested that in the studies related to sun-weather-climate, one can look specifically for the contribution of such periodic oscillations and significant correlations rather than analyzing the original time series as such. Perhaps greater insight could be obtained if concurrent time series which are proxy for climate features are also analyzed in a similar fashion to establish the reality or otherwise of the inter-relationships. Our attempted extrapolation of the time series up to 2015 shows that there is a tendency for the length to increase, albeit over a small range, up to about 2010.

Acknowledgements

We are thankful to Mr. Ronaldo Marins de Carvalho for his assistance in digitizing the data. GKR expresses his thanks to CNPq, Brazil for the award of a fellowship to work as a Visiting Professor. He also thanks the Director, Observatório Nacional for the facilities provided to carry out this research.

References


